

Exam II: Discrete Math, MTH 213, Fall 2017

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$\frac{55}{55}$

M. Said (Perfect score, second time in a row)

QUESTION 1. Assume there are 550 persons in the main building of AUS. Then

(i) There are at least n persons who were born in the same month and on the same day of the week. What is the maximum value of n that we all are sure about?

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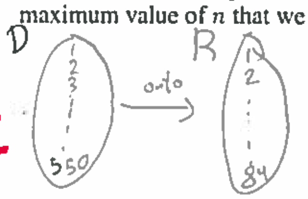

it's an onto function where $|R| < |D|$ and we assume fare sharing $\Rightarrow$ by Pigeonhole Principle we have at least $n = \lceil \frac{550}{84} \rceil = \boxed{7}$ persons we are sure abo[ut] that were born on the same day in the same month

(ii) There must exist a day of the week such that no more than m persons were born on that day. what is the maximum value of m ?(hint: THINK! not difficult)

2

if all students were born on the same day of the week the we can have maximum $\boxed{m = 550}$ Persons who were born on that day.

QUESTION 2. (i) 26 distinct numbers were chosen randomly. Then there are at least n numbers out of the chosen numbers that have the same unit digit. What is the maximum value of n that we all are sure about?

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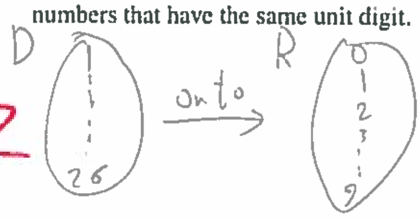

it's an onto function where $|R| < |D|$ and assuming fare sharing $\Rightarrow$ by Pigeonhole principle we have at least $n = \lceil \frac{26}{10} \rceil = \boxed{3}$ numbers that have the same unit digit

(ii) Assume 302 persons were in a party. Assume that the party started at 8pm and it ended at 2am. Then there are at least n persons in the party of the same sex. Find the maximum value of n that we all are sure about? (Hint: Smile!)

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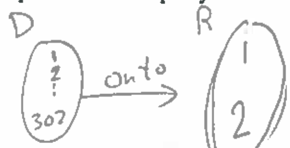

it's an onto function where $|R| < |D|$ and assuming fare sharing $\Rightarrow$ by Pigeonhole Principle we have at least $n = \lceil \frac{302}{2} \rceil = \boxed{151}$ Persons of the same sex

QUESTION 3. We have 7 holes labeled from 1 to 7 and we have 4 balls (red, blue, green, yellow). We need to put each ball in one hole. Find all possible ways?

2 $7C4 \times 4P4$

QUESTION 4. An electrical panel has six switches. How many ways can the switches be positioned up or down if four switches must be up and 2 switches must be down. (note order not important)

2 $6C4 \times 2C2$

QUESTION 5. How many 4-digit even numbers greater than 400 can be formed using the digits 1, 2, 3, 4, 5, and 6?

2

$X = \underline{x_1}\ \underline{x_2}\ \underline{x_3}\ \underline{x_4}$

$X = 6C1 \times 6C1 \times 6C1 \times 3C1 = 648$

1 _ _ _
2 _ _ _
3 _ _ _
4 _ _ _
5 _ _ _
6 _ _ _

QUESTION 6. If there are 3 buses and 4 cars, how many ways can the vehicles park in a line such that cars and buses alternate positions?(note order is important)

2

$3P3 \times 4P4$

QUESTION 7. (i) the Mickey-function $\frac{x^{0.5}+3x^{5/2}-5}{x+7}$ is $\Theta(X^{\frac{3}{2}})$ and it is $O(X)^{\frac{3}{2}}$

2

(ii) The Mickey-polynomial $\sqrt{x}(x^3 - x^{11/2} + 7)$ is $\Theta(X^6)$ and it is $O(X)^6$

2

$X^{\frac{7}{2}} - X^6 + 7$

QUESTION 8. Consider the following Algorithm segment. Find the exact number of additions, multiplications, and subtractions that will be performed when the algorithm is executed. Then find the order of the Algorithm segment.

$m = 7; s = 0$

For $k := 4$ *to* $n + 1$ → $n+1-4+1 = n-2$

For $i := 1$ *to* 10 → $10-1+1 = 10$

$s = s + m^3 + i - k^2$ 6

next i

$L = k + 2 * s - 6$ 3

next k

4

for $k = 4$ ...... $k = n+1$

$(10 \times 6) + 3$ $(10 \times 6) + 3$

$$\frac{(n-2)(63+63)}{2} = \frac{126n - 252}{2} = \boxed{63n - 126}$$

the number of multiplications, subtractions and addiors

the code is of order $\Theta(n)$ and $O(n)$

QUESTION 9. Given string1, say S_1 : 1 1 0 1 0 0 and string2, say S_2 : 1 1 1 1 0 1

a) Find $S_1 \wedge S_2$

110100

b) Find $S_1 \vee S_2$

111101

4

c) Find $S_1 \oplus S_2$

001001

d) Find $\neg S_1 \vee S_2$

$\neg S_1$ = 001011 $\vee$ 111101 = 111111

QUESTION 10. Convince me that $S_1 \wedge (S_2 \rightarrow S_3) \equiv (S_1 \wedge \neg S_2) \vee (S_1 \wedge S_3)$

S_1	S_2	S_3	$(S_2 \rightarrow S_3)$	$(S_1 \wedge \neg S_2)$	$(S_1 \wedge S_3)$	$S_1 \wedge (S_2 \rightarrow S_3)$	$(S_1 \wedge \neg S_2) \vee (S_1 \wedge S_3)$
0	0	0	1	0	0	0	0
0	0	1	1	0	0	0	0
0	1	0	0	0	0	0	0
0	1	1	1	0	0	0	0
1	0	0	1	1	0	1	1
1	0	1	1	1	1	1	1
1	1	0	0	0	0	0	0
1	1	1	1	0	1	1	1

identical

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QUESTION 11. Given a sequence $\{b_n\}_{n=0}^{\infty}$, where $b_0 = 2, b_1 = 2$, and $b_n = 6b_{n-1} - 9b_{n-2} + \sqrt{n} + 1$. Find a general formula for b_n.

QUESTION 12. Write down T or F

(i) If $\exists x \in N^*$ such that $x + 4 = 3$, then $x^2 + 2 = 20$ T

(ii) If $\exists x \in R$ such that $x^2 + 4 = 8$, then $x - 2 = 0$ F

(iii) If every rectangle is a square, then every ellipse is a circle. T

$\frac{3}{3}$

QUESTION 13. (i) Is the sequence 3, 3, 3, 3 a graphical? If yes, then draw such graph, and then find the girth and the diameter of the graph.

3 3 3 3 ≡ 2 2 2 ≡ 1 1 ⇒ 1 this non-increasing sequence is graphical

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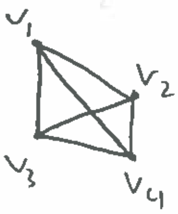


girth = 3 , diam = 1

(ii) Is the sequence 5, 4, 4, 4, 4, 4 a graphical? If yes, then find number of all edges of such graph.

3

5 4 4 4 4 4 ≡ 3 3 3 3 3 ≡ 3 2 2 2 ≡ 1 1 1

≡ 1 0 ≡ $\underline{-1}$ ⇒ this sequence is not graphical

QUESTION 14. Let D be a graph with vertex set $V = \{0, 1, 2, ..., 8\}$. Assume that every two distinct vertices, say x, y, are connected by an edge if and only if $3 \mid (x + y)$ (i.e., 3 is a factor of $x + y$). By drawing such graph, convince me that D is disconnected. Convince me that D consists of two components, one component is K_m for some m and the other component is $K_{n,n}$ for some n. Find the values of m and n.

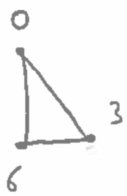
3: 0 − 3, 1 − 2

6: 0 − 6, 1 − 5, 2 − 4

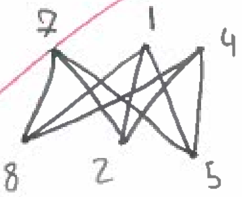
9: 1 − 8, 2 − 7, 3 − 6, 4 − 5

12: 4 − 8, 5 − 7

15: 7 − 8



$K_3 \Rightarrow m = 3$



$K_{3,3} \Rightarrow n = 3$

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